

Masses and Mixing of Quarks and Leptons in Product-Group Unification

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We discuss a supersymmetric unified model based on a product gauge group $SU(5) \times SU(5) \times SU(5)$, where the gauge symmetry breaking is achieved without the adjoint or higher-dimensional Higgs field, and the doublet-triplet splitting in the Higgs masses is realized by the use of the discrete symmetry. In this article we present an explicit model for realistic fermion masses with the discrete symmetries $Z_7 \times Z_2$. It is shown that all the observed masses and mixing angles for quarks and leptons, including neutrinos, are well described by the breaking of the symmetries imposed in the model. Especially, the maximal and large mixing angles in the atmospheric and solar neutrino oscillations are obtained as the most preferred values, and the typical value of the neutrino mixing element U_{e3} is 0.1–0.3. We also point out the non-trivial relations among the μ -parameter for the Higgs mass, the charged fermion hierarchies, and the neutrino masses. These relations suggest that the scale of μ is of order of the weak scale.

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I. INTRODUCTION

The unification of strong, weak and electromagnetic interactions is a very attractive idea [1]. The gauge coupling unification supports strongly this grand unified theories (GUTs) with a supersymmetry at low energy. The simplest model is based on an $SU(5)$ group [2]. In spite of the theoretical appeal, there have been several puzzles in construction of supersymmetric GUT models: The first puzzle is the Higgs sector. In general, the very complicated and innovative Higgs sector should be considered. The GUT gauge breaking usually requires the Higgs field(s) in the adjoint or higher-dimensional representation(s). In particular, the most annoying difficulty is the mass splitting between Higgs weak-doublets and color-triplets. Various mechanisms for this problem have been proposed [3, 4, 5, 6, 7].

The second one concerns the matter sector. Quarks and leptons are unified into irreducible representation(s) of the GUT group. This matter unification ensures the electric charge quantization, but also predicts a promising mass relation, $m_b = m_\tau$, in the simplest $SU(5)$ model. However, it predicts also the unwanted mass relations, $m_s = m_\mu$ and $m_d = m_e$, which are badly broken in nature. Moreover, recent neutrino experiments [8, 9, 10, 11] reveal that the neutrinos own non-zero masses which are much smaller than the electron mass, and remarkably that the mixing angles in the atmospheric and solar neutrino oscillations are both large. The rest angle is only limited by experiment [12]. We then have to explain small and large mixing angles in the quark-lepton sector at the same time. The observed fermion properties may also require the GUT models to be complicated.

One way to evade these issues is to give up the fundamental assumption – a simple group –, *i.e.*, to go into the product group like $G \times G$ or $G \times G \times G$, where G denotes the GUT gauge groups [13, 14, 15, 16, 17, 18, 19, 20, 21]. As pointed out in Ref. [13] the class of these models,

called the product-group unification, still preserves the nice features of the GUTs. Namely, the standard gauge group is contained in the diagonal subgroup, and hence the coupling unification is realized at tree-level. In addition, quarks and leptons are unified in each GUT group G , and the electric charge quantization is also explained.

There are three major advantages in the product-group unification: First, the breaking of gauge symmetry is accomplished without introducing the Higgs fields in the adjoint or higher-dimensional representations [13]. This seems very promising for the connection to the string theory. In fact, it is known that there is no state in the adjoint or higher representation in the string spectrum with the affine level one [22]. Second, the doublet-triplet problem may receive a natural solution by the use of the discrete symmetry [13, 17, 19, 20, 21].¹ Finally, we may gain important insights to the flavor physics. It is beyond the scope of the usual GUTs (as well as the standard model) to understand the origin of the hierarchical structure in the Yukawa couplings, which are essentially free parameters. On the other hand, the mass hierarchy between different flavors may be explained by the breaking of the product group when they belong to different GUT group factors [14]. This illustrates the idea of Refs. [24, 25] in the unified theories. Moreover, the discrete symmetry for the doublet-triplet splitting can be utilized as the horizontal flavor symmetry, which is also crucial for the suppression of the dimension-five operators to avoid a rapid proton decay [14, 19, 20, 21].

In this article, we present an explicit model for realistic fermion masses based on the product group $SU(5) \times SU(5) \times SU(5)$. Especially, we emphasize on the neutrino properties. Although the similar analyses have

¹ In the different class of models with a product group the unbroken R -symmetry is used for the doublet-triplet splitting [23].

been done in Refs. [14, 19], the model in Ref. [14] leads to the small angles for neutrino mixing, and in Ref. [19] the neutrino physics has not been discussed. Our model is constructed by imposing the discrete symmetries $Z_7 \times Z_2$, where the Z_7 symmetry is introduced to solve the doublet-triplet splitting. The observed masses and flavor mixing of quarks and leptons, including neutrinos, are well described by the breaking of the product gauge group together with these discrete symmetries. It is shown that the Z_7 breaking is crucial for generating (i) an effective μ -term for the Higgs doublets, (ii) the charged fermion mass hierarchies, and (iii) the Majorana masses for right-handed neutrinos.² We point out that the μ -parameter at the weak scale is suggested from not only the observed masses of charged fermions, but also the neutrino masses indicated by the oscillation experiments.

This article is organized as follows: in the next section, we first present our model based on a product group $SU(5) \times SU(5) \times SU(5)$. In particular, the gauge symmetry breaking and the doublet-triplet splitting are discussed. In Sec. III we construct the fermion mass matrices from the invariance under the gauge symmetry as well as the discrete symmetries. The masses and mixing angles of quarks and leptons, including neutrinos, are discussed in Sec. IV. We compare the prediction of our model with the experimental data. In Sec. V the impact on the leptogenesis of our model is considered. Finally, Sec. VI contains our conclusion.

II. $SU(5)_1 \times SU(5)_2 \times SU(5)_3$ MODEL

A. Gauge Symmetry Breaking

In this article we will investigate a supersymmetric model based on a product group:

$$G = SU(5)_1 \times SU(5)_2 \times SU(5)_3. \quad (1)$$

One of the advantages in this class of models based on product groups is that the gauge symmetry breaking can be achieved without introducing Higgs fields in the adjoint or higher-dimensional representations. In fact, as pointed out in Ref. [13], the bifundamental fields are sufficient for this task. We introduce here three vector-like pairs of the bifundamental fields, (T_1, \bar{T}_1) , (D_1, \bar{D}_1) and (V_2, \bar{V}_2) . Their gauge charges are presented in Table I. Although $T_1(\bar{T}_1)$ and $D_1(\bar{D}_1)$ have the same gauge charges, but they carry different charges under discrete symmetries. The breaking of the group G into the standard model group $SU(3) \times SU(2) \times U(1)$ is achieved by the

TABLE I: Field contents of the model.

	$SU(5)_1$	$SU(5)_2$	$SU(5)_3$	Z_7	Z_2
T_1	5	1	5*	1	−
\bar{T}_1	5*	1	5	6	−
D_1	5	1	5*	2	−
\bar{D}_1	5*	1	5	5	+
V_2	1	5	5*	0	−
\bar{V}_2	1	5*	5	0	−
H	1	1	5	2	+
\bar{H}	5*	1	1	4	−
10_1	10	1	1	1	+
10_2	1	10	1	6	+
10_3	1	1	10	6	+
5_1^*	1	5*	1	3	+
5_2^*	1	1	5*	2	+
5_3^*	1	1	5*	2	+
1_1	1	1	1	2	+
1_2	1	1	1	3	+
1_3	1	1	1	3	+

vacuum expectation values (VEVs) of these fields:

$$\langle T_1 \rangle = \langle \bar{T}_1 \rangle = \text{diag}(T_1, T_1, T_1, 0, 0), \quad (2a)$$

$$\langle D_1 \rangle = \langle \bar{D}_1 \rangle = \text{diag}(0, 0, 0, D_1, D_1), \quad (2b)$$

$$\langle V_2 \rangle = \langle \bar{V}_2 \rangle = \text{diag}(T_2, T_2, T_2, D_2, D_2), \quad (2c)$$

where we take all the VEVs are real and positive.³ These bifundamental fields play also an important rôle for fermion masses and mixing. As we will see in Sec. IV, the observational data suggest the scale of the VEVs is typically of the order of $0.1 M_*$ (M_* is the cut-off scale which is taken as the reduced Planck scale in this analysis). Below these scales, we obtain the minimal supersymmetric standard model.

The above VEVs might be determined by a superpotential for the bifundamental fields. Such a superpotential should avoid the appearance of the massless Nambu-Goldstone fields. The standard model group is embedded in a diagonal $SU(5)$ group of G , and the gauge coupling unification is ensured at the leading order, although we have to include the threshold corrections around the G breaking scale. In this article, we will not discuss these issues, but we assume the VEVs as in Eqs. (2).

B. Doublet-Triplet Splitting

One of the serious difficulties in construction of GUT models is the doublet-triplet splitting in Higgs masses.

² The relation between the large Majorana masses for right-handed neutrinos and the breaking of the horizontal global symmetry has been discussed in Ref. [26].

³ We use sometimes the same letters for the superfields and for their VEVs.

There exist two weak-doublets of Higgs fields, H_u and H_d , in the minimal supersymmetric standard model, where H_u and H_d are superfields whose VEVs give masses to up- and down-type quarks, respectively. We incorporate these Higgs doublets by introducing

$$H = (G_u, H_u)^T, \quad \bar{H} = (G_d^c, H_d)^T, \quad (3)$$

together with color-triplet Higgs G_u and G_d^c . Here H and \bar{H} are $\mathbf{5}$ and $\mathbf{5}^*$ -plets of $SU(5)_3$ and $SU(5)_1$, respectively. It should be emphasized that the gauge symmetry G forbids an invariant mass term for H and \bar{H} , however, the terms $\bar{H}T_1H$ and $\bar{H}D_1H$ are allowed in the superpotential. Although the former term is desirable to give a heavy mass to the color-triplet Higgs, the latter should be forbidden to leave the weak-doublet Higgs massless below the G breaking scale. How can we distinguish the weak-doublet Higgs from the color-triplet Higgs in a natural way?

The doublet-triplet splitting can be realized by using a discrete symmetry in the framework of the product-group unification [13, 14, 17, 19, 20, 21]. Here we shall follow the discussion of Ref. [21]. Such a symmetry can be constructed from a usual discrete symmetry Z_N and a discrete ‘‘hypercharge’’ subgroup of $U(1)_1 \subset SU(5)_1$ or $U(1)_3 \subset SU(5)_3$. In the present model we choose that of $U(1)_1$ for example. Its element (combining with the Z_N transformation) is parametrized as

$$g_1 = \text{diag}(\alpha^{-1}, \alpha^{-1}, \alpha^{-1}, \alpha^{\frac{N+3}{2}}, \alpha^{\frac{N+3}{2}}), \quad (4)$$

where α is an N -th root of unity. For the later purpose we consider only an odd integer N . We assign the Z_N charges for the bifundamental fields as

$$\begin{aligned} T_1 &\rightarrow \alpha T_1, & D_1 &\rightarrow \alpha^{-\frac{N+3}{2}} D_1, \\ \bar{T}_1 &\rightarrow \alpha^{-1} \bar{T}_1, & \bar{D}_1 &\rightarrow \alpha^{\frac{N+3}{2}} \bar{D}_1, \end{aligned} \quad (5)$$

and V_2 and \bar{V}_2 carry zero charge. Then, it turns out that the VEVs shown in Eq. (2) preserve a \tilde{Z}_N symmetry which is the combination of Z_N and g_1 .

This unbroken discrete symmetry \tilde{Z}_N can realize the doublet-triplet splitting. We assign the Z_N charges for H and \bar{H} :

$$H \rightarrow \alpha^{n_H} H, \quad \bar{H} \rightarrow \alpha^{-n_H-1} \bar{H}, \quad (6)$$

with an integer n_H ($= 0, \dots, N-1$), and the transformations under the \tilde{Z}_N of each components are given by

$$\begin{aligned} H_u &\rightarrow \alpha^{n_H} H_u, & G_u &\rightarrow \alpha^{n_H} G_u, \\ H_d &\rightarrow \alpha^{-n_H-\frac{N+5}{2}} H_d, & G_d^c &\rightarrow \alpha^{-n_H} G_d^c. \end{aligned} \quad (7)$$

We can see that the \tilde{Z}_N symmetry, which is left after the G breaking, discriminates between the weak-doublet H_d and the color-triplet G_d^c as long as $N \neq 5$. In fact, if it is the case, the term $\bar{H}D_1H$ will be absent in the superpotential, while the following term is still allowed

$$W \supset \bar{H}T_1H, \quad (8)$$

which generates a heavy mass of $T_1 \sim 0.1M_*$ to the color-triplet Higgs.

The exact \tilde{Z}_N symmetry leads to no mass term for the weak-doublet Higgs in the superpotential. The so-called μ -term which is needed for the electroweak symmetry breaking is generated only by its breaking. For this purpose we introduce a gauge-singlet field Φ which carries a Z_N charge $+1$ and includes the following term (henceforth we take the cut-off scale M_* to be one)

$$W \supset c_\mu \bar{H}D_1H \Phi^{\frac{N+5}{2}}, \quad (9)$$

where c_μ is a constant and we set $c_\mu = 1$ for simplicity. The effective μ -parameter then becomes⁴

$$\mu = D_1 \Phi^{\frac{N+5}{2}}. \quad (10)$$

For examples, when $\mu = 100$ GeV and $D_1 = 0.1$, the VEV of Φ is given by $\Phi = 1.4 \times 10^{-4}$ ($N = 3$), 2.7×10^{-3} ($N = 7$) and 6.3×10^{-3} ($N = 9$).

C. Matter Fields

Now we turn to discuss the matter fields in our model. In the simplest $SU(5)$ model one family of quarks and leptons are grouped into a $\mathbf{10}$ -plet and a $\mathbf{5}^*$ -plet. Similarly, we introduce the matter fields as $\mathbf{10}$ -plets and $\mathbf{5}^*$ -plets of an $SU(5)$ factor of G , which guarantees the electric charge quantization. Concretely speaking, 10_i , 5_i^* and 1_i ($i = 1, 2, 3$) are introduced as the matter fields. The charges under the gauge group G can be also found in Table I. First, each 10_i is allocated to a $\mathbf{10}$ -plet of $SU(5)_i$, respectively. Then, the gauge charges of H and \bar{H} results in a unique choice for the distribution of 5_i^* to cancel the gauge anomalies. Namely, one of 5_i^* , say 5_1^* , should be a $\mathbf{5}^*$ -plet of $SU(5)_2$ and the rest, 5_2^* and 5_3^* , should be $\mathbf{5}^*$ -plets of $SU(5)_3$. Finally, three gauge-singlet fields 1_i (right-handed neutrinos) are introduced to generate neutrino masses.

This matter content leads to interesting features of the model: First, apart from the Majorana masses for right-handed neutrinos, the gauge symmetry G allows a Yukawa coupling only for top-quark. All other Yukawa couplings are induced effectively by the G breaking and are suppressed by some powers of VEVs of the bifundamental fields. This accounts for, at the first approximation, a large top-quark mass and smaller masses for the other fermions.

Second, our model offers naturally the so-called ‘‘lopsided family structure’’ [27, 28, 29, 30]. The distribution of 10_i under G is different from that of 5_i^* to cancel the gauge anomalies. This may be the reason why there is a

⁴ Although the effects of the supersymmetry breaking may also give an additional contribution to μ , it can be neglected when the supersymmetry breaking field carries the Z_N charge zero.

difference in the mass hierarchies between up- and down-type quarks. More importantly, the same gauge charge between 5_2^* and 5_3^* may ensure a large ν_μ - ν_τ mixing angle confirmed in the atmospheric neutrino oscillation.⁵

III. FERMION MASS MATRICES

In this section we present fermion mass matrices in our model. As discussed above, the product gauge group G is considered as the flavor symmetry. The discrete symmetry Z_N for the doublet-triplet splitting is also important for fermion masses. Since the bifundamental fields, H and \overline{H} carry non-trivial Z_N charges, the matter fields should also transform non-trivially under the Z_N . Some Yukawa couplings are forbidden and induced effectively by its breaking together with some power of the VEV of the field Φ . This means that the Z_N symmetry is crucial not only for the doublet-triplet splitting but also for the hierarchies in fermion masses.

In this article we present the model based on the Z_7 symmetry ($N = 7$). This is because, first, the doublet-triplet splitting is realized only if $N \neq 5$. Second, when $N = 3$, the VEV of Φ is too small to explain the charged fermion masses [see the discussion below Eq. (10)]. Finally, the larger N , *i.e.*, the larger VEV of Φ , is disfavored since it induces too small neutrino masses. In the following, therefore, we will construct the fermion mass matrices based on (i) the gauge symmetry G and (ii) the Z_7 symmetry for the doublet-triplet splitting. Further, we will impose (iii) an additional Z_2 symmetry. The reason for its inclusion will be clear later.

A. Mass Matrices for Charged Fermions

Let us first consider the Yukawa terms for charged fermions, and we will discuss neutrino masses later. We define the Yukawa couplings by

$$W = Y_{uij} H u_i^c u_j + Y_{dij} \overline{H} d_i^c d_j + Y_{eij} \overline{H} e_i^c e_j, \quad (11)$$

where u_i and u_i^c are the left- and right-handed up-type quarks, *etc.* Without taking into account the discrete symmetries, the gauge charges of matter fields give the

effective Yukawa couplings:

$$Y_u = \begin{pmatrix} D_1 & \overline{T}_1^2 \overline{D}_2 \overline{T}_2 & \overline{T}_1^2 \\ \overline{D}_1 \overline{T}_1 \overline{T}_2^2 & D_2 & \overline{T}_2^2 \\ \overline{D}_1 \overline{T}_1 & \overline{D}_2 \overline{T}_2 & 1 \end{pmatrix}, \quad (12a)$$

$$Y_d = \begin{pmatrix} \overline{T}_1 T_2 & D_1 \overline{D}_2 & D_1 T_2 \\ \overline{T}_1 & D_1 \overline{D}_2 \overline{T}_2 & D_1 \\ \overline{T}_1 & D_1 \overline{D}_2 \overline{T}_2 & D_1 \end{pmatrix}, \quad (12b)$$

$$Y_e^T = \begin{pmatrix} \overline{D}_1 D_2 & D_1 \overline{D}_2 & D_1 D_2 \\ \overline{D}_1 & D_1 \overline{D}_2^2 & D_1 \\ \overline{D}_1 & D_1 \overline{D}_2^2 & D_1 \end{pmatrix}, \quad (12c)$$

where the bifundamental fields should be considered as their diagonal components which have non-zero VEVs (cf. $\langle T_2 \rangle = \langle \overline{T}_2 \rangle = T_2$ and $\langle D_2 \rangle = \langle \overline{D}_2 \rangle = D_2$). These results are obtained at the leading order of the power expansion in the bifundamental fields. As we will study in Sec. IV, all VEVs of the bifundamental fields turn out to be of order 0.1 (in the unit $M_* = 1$), and hence the leading terms are sufficient for our discussion.

It should be noted that we have omitted dimensionless coefficients in front of each element in the above Yukawa couplings. In principle, we can take them as any complex numbers. Here, however, following to the idea of the Froggatt-Nielsen mechanism [31], we assume that these unknown coefficients are of order one, and thus the hierarchy in the Yukawa couplings are generated by the breaking of the product gauge group G (as well as the discrete symmetries, see below). Therefore, we should keep in mind that our analysis is of order of magnitude wise and receives the uncertainty from our ignorance to determine these coefficients.

As mentioned above, the Z_7 symmetry for the doublet-triplet splitting gives rise an additional hierarchical structure in the Yukawa couplings (12). Due to the smallness of the suppression factor, $\Phi \sim 10^{-3}$, we have to choose carefully the Z_7 charges for matter fields as well as Higgs fields in order to obtain realistic fermion mass matrices. Notice that the Z_7 charges for bifundamental fields have already determined in Sec. II.

Let us start with the charge assignment of the third family fermions. For a large top-quark mass we require the following term in the superpotential without any suppression factor of Φ

$$W \supset H 10_3 10_3, \quad (13)$$

and also we need the term,

$$W \supset \overline{H} D_1 10_3 5_3^*, \quad (14)$$

to explain bottom and tau masses. Moreover, the neutrino mass would come from the term, $W \supset H H 5_3^* 5_3^*$, which, unfortunately, induces too small neutrino mass scale compared with the neutrino oscillation experiments. From this reason, we require the following term as a con-

⁵ The same structure of the matter and Higgs fields (but the different structure of the bifundamental fields) can be found in Ref. [19]. However, the different mass matrices are induced and also the implication to the neutrino properties are not discussed there.

sequence of the seesaw mechanism [32, 33].⁶

$$W \supset \frac{1}{\Phi} H H 5_3^* 5_3^*. \quad (15)$$

With these three requirements we may choose the Z_N charges as

$$H : 2, \quad \bar{H} : 4, \quad 10_3 : 6, \quad 5_3^* : 2. \quad (16)$$

The Z_7 charges for other matter fields are found as follows: We find from m_μ and m_s that the charges for 10_2 and 5_2^* should be same as 10_3 and 5_3^* , respectively. Of course, we have to avoid too small electron mass when the charges of 10_1 and 5_1^* are determined. The results are

$$10_1 : 1, \quad 10_2 : 6, \quad 5_1^* : 3, \quad 5_2^* : 2. \quad (17)$$

These charges are also listed in Table I. With this Z_7 charge assignment the Yukawa couplings take the forms:

$$Y_u = \begin{pmatrix} \Phi D_1 & \bar{T}_1^2 \bar{D}_2 \bar{T}_2 & \bar{T}_1^2 \\ \Phi \bar{D}_1 \bar{T}_1 \bar{T}_2^2 & D_2 & \bar{T}_2^2 \\ \Phi \bar{D}_1 \bar{T}_1 & \bar{D}_2 \bar{T}_2 & 1 \end{pmatrix}, \quad (18a)$$

$$Y_d = \begin{pmatrix} \bar{T}_1 T_2 & \Phi^6 D_1 \bar{D}_2 & \Phi^6 D_1 T_2 \\ \Phi \bar{T}_1 & D_1 \bar{D}_2 \bar{T}_2 & D_1 \\ \Phi \bar{T}_1 & D_1 \bar{D}_2 \bar{T}_2 & D_1 \end{pmatrix}, \quad (18b)$$

$$Y_e^T = \begin{pmatrix} \Phi \bar{D}_1 D_2 & \Phi^6 D_1 \bar{D}_2 & \Phi^6 D_1 D_2 \\ \Phi^2 \bar{D}_1 & D_1 \bar{D}_2^2 & D_1 \\ \Phi^2 \bar{D}_1 & D_1 \bar{D}_2^2 & D_1 \end{pmatrix}. \quad (18c)$$

Comparing with Eqs. (12) there appear suppression factors in some Yukawa couplings.

We find, however, the Yukawa couplings in Eqs. (18) predict wrong mass ratios, m_u/m_t , m_c/m_t and m_d/m_b , which conflict with the observation by orders of magnitude. The present model calls for an additional discrete symmetry Z_2 to correct these mass relations. We assign non-trivial charges only for the bifundamental and Higgs fields. Since all matter fields carry even parity of Z_2 , this Z_2 symmetry should not be considered as a usual horizontal flavor symmetry. Notice that this Z_2 symmetry does not affect the mass term for the color-triplet Higgs in Eq. (8) and the effective μ -term in Eq. (9). From the Z_2 symmetry the parity-odd components in the Yukawa couplings receive a suppression factor of $(D_1 \bar{D}_1) = \text{Tr}(D_1 \bar{D}_1) = 2D_1^2$. In the following analysis we neglect this factor of two which is beyond our approximation.

Finally, the effective Yukawa couplings in the model are given in the forms

$$Y_u = \begin{pmatrix} \Phi D_1 (D_1 \bar{D}_1) & \bar{T}_1^2 \bar{D}_2 \bar{T}_2 & \bar{T}_1^2 \\ \Phi \bar{D}_1 \bar{T}_1 \bar{T}_2^2 (D_1 \bar{D}_1) & D_2 (D_1 \bar{D}_1) & \bar{T}_2^2 \\ \Phi \bar{D}_1 \bar{T}_1 (D_1 \bar{D}_1) & \bar{D}_2 \bar{T}_2 & 1 \end{pmatrix}, \quad (19a)$$

$$Y_d = \begin{pmatrix} \bar{T}_1 T_2 (D_1 \bar{D}_1) & \Phi^6 D_1 \bar{D}_2 & \Phi^6 D_1 T_2 \\ \Phi \bar{T}_1 (D_1 \bar{D}_1) & D_1 \bar{D}_2 \bar{T}_2 & D_1 \\ \Phi \bar{T}_1 (D_1 \bar{D}_1) & D_1 \bar{D}_2 \bar{T}_2 & D_1 \end{pmatrix}, \quad (19b)$$

$$Y_e^T = \begin{pmatrix} \Phi \bar{D}_1 D_2 & \Phi^6 D_1 \bar{D}_2 (D_1 \bar{D}_1) & \Phi^6 D_1 D_2 (D_1 \bar{D}_1) \\ \Phi^2 \bar{D}_1 (D_1 \bar{D}_1) & D_1 \bar{D}_2^2 & D_1 \\ \Phi^2 \bar{D}_1 (D_1 \bar{D}_1) & D_1 \bar{D}_2^2 & D_1 \end{pmatrix}. \quad (19c)$$

By putting the non-zero VEVs, the matrices for charged fermions are obtained as follows:

$$M_u = H_u \begin{pmatrix} \Phi D_1^3 & T_1^2 D_2 T_2 & T_1^2 \\ \Phi D_1^3 T_1 T_2^2 & D_1^2 D_2 & T_2^2 \\ \Phi D_1^3 T_1 & D_2 T_2 & 1 \end{pmatrix}, \quad (20a)$$

$$M_d = H_d \begin{pmatrix} D_1^2 T_1 T_2 & \Phi^6 D_1^3 D_2 & \Phi^6 D_1^3 T_2 \\ \Phi T_1 & D_1 D_2 T_2 & D_1 \\ \Phi T_1 & D_1 D_2 T_2 & D_1 \end{pmatrix}, \quad (20b)$$

$$M_e^T = H_d \begin{pmatrix} \Phi D_1 D_2 & \Phi^6 D_1 D_2 & \Phi^6 D_1 T_2 \\ \Phi^2 D_1^3 & D_1 D_2^2 & D_1 \\ \Phi^2 D_1^3 & D_1 D_2^2 & D_1 \end{pmatrix}. \quad (20c)$$

It is rather important to mention that the second and third rows in M_d and M_e^T have the same structure, since there is no symmetry which distinguishes between 5_2^* and 5_3^* . In this analysis, however, we assume that unknown coefficients in front of 5_2^* and 5_3^* are different. Note that some coefficients are related of each other due to $\text{SU}(5)$ gauge groups. For example, the 3-3 components in M_d and M_e are identical.

B. Neutrino Mass Matrices

Now, we turn to discuss mass matrices for neutrinos. We define the Dirac and the Majorana mass matrices by

$$W = \nu_i^c (M_D)_{ij} \nu_j + \frac{1}{2} \nu_i^c (M_N)_{ij} \nu_j^c. \quad (21)$$

Since right-handed neutrinos $\nu_i^c = 1_i$ are totally gauge singlets, the gauge group G tells nothing about the structure of their Majorana masses. On the other hand, they can carry non-trivial charges under the discrete Z_7 symmetry for the doublet-triplet splitting, and the origin of the Majorana masses and their structure may be explained by the Z_7 breaking. Moreover, this fact leads to the relation between the μ -parameter for the Higgs mass and the neutrino masses in our model. As we will

⁶ The same assignment of the Z_7 charges might induce the operator $W \supset \Phi^6 H H 5_3^* 5_3^*$, however, our model is not such a case as we will show in Eq. (47).

show in the next section, this relation fits very well to the observations.

We assign here the Z_7 charges for the right-handed neutrinos in the following way:

$$1_1 : 2, \quad 1_2 : 3, \quad 1_3 : 3. \quad (22)$$

Then, the Majorana mass matrix becomes:

$$M_N = \Phi \begin{pmatrix} \Phi^2 & \Phi & \Phi \\ \Phi & 1 & 1 \\ \Phi & 1 & 1 \end{pmatrix}. \quad (23)$$

The Z_7 breaking generates the Majorana masses M_i ($i = 1, 2, 3$) as

$$M_1 \sim \Phi^3, \quad M_{2,3} \sim \Phi. \quad (24)$$

This shows that right-handed neutrinos acquire super-heavy masses due to $\Phi = \mathcal{O}(10^{-3})$, hence they offer a natural setup for the seesaw mechanism [32].

The Z_7 charges for right-handed neutrinos in Eq. (22) are also important to construct the Dirac mass matrix M_D . These charges, together with the gauge symmetry G and the discrete symmetry Z_2 , generates M_D in the form

$$M_D = H_u \begin{pmatrix} D_1^2 D_2 & \Phi & \Phi \\ \Phi^6 D_1^2 D_2 & 1 & 1 \\ \Phi^6 D_1^2 D_2 & 1 & 1 \end{pmatrix}. \quad (25)$$

Here the suppression factor D_1^2 appears in the first row due to the Z_2 symmetry. Similar to the mass matrices for down-type quarks and charged leptons in Eqs. (20); the second and the third rows in M_D take the same structure because 5_2^* and 5_3^* carry the same charges under all the symmetries. These facts lead to the large flavor mixing between the second and third families in the left-handed leptons and right-handed down-quarks. Furthermore, we should note that the 2-1 and 3-1 elements in M_D are extremely suppressed and essentially zeros. These missing elements give us the desired hierarchy in the neutrino mixing angles. We will give these particulars in the next section.

To summarize this section, we have constructed the mass matrices for all quarks and leptons including neutrinos based on the product group G , the Z_7 symmetry for the doublet-triplet splitting, and the additional Z_2 symmetry. The structure of the mass matrices arise from the breaking of these symmetries under our assumption that all the unknown coupling constants be of order one. In the following section, we will compare the obtained results with the observation.

IV. FERMION MASSES AND MIXING

We are now at the point to evaluate masses and mixing angles of quarks and leptons in our model. The fermion

mass matrices are shown in Eqs. (20), (23) and (25). Apart from unknown coefficients of order one, they are all determined by the following six parameters:

$$T_1, \quad D_1, \quad T_2, \quad D_2, \quad \Phi, \quad \tan \beta, \quad (26)$$

where $\tan \beta = H_u/H_d$, thus we expect rather non-trivial predictions between masses and mixing angles.

A. Charged Fermions

We first discuss masses and flavor mixing of charged fermions. From Eqs. (20) the third family masses are found to be

$$m_t \sim H_u, \quad m_b \simeq m_\tau \sim H_d D_1, \quad (27)$$

and we also find the following mass hierarchies:

$$\frac{m_c}{m_t} \sim D_1^2 D_2, \quad \frac{m_s}{m_b} \sim D_2 T_2, \quad \frac{m_\mu}{m_\tau} \sim D_2^2, \quad (28a)$$

$$\frac{m_u}{m_t} \sim \Phi D_1^3, \quad \frac{m_d}{m_b} \sim D_1 T_1 T_2, \quad \frac{m_e}{m_\tau} \sim \Phi D_2. \quad (28b)$$

Note that masses in the first family can be estimated in a rather solid way since the off-diagonal elements in the first column of M_u and in the first row of M_d and M_e^T are strongly suppressed. On the other hand, the elements of the CKM matrix are found as

$$|V_{us}| \sim \frac{\Phi T_1}{D_1 D_2 T_2}, \quad |V_{cb}| \sim D_2 T_2, \quad |V_{ub}| \sim \frac{\Phi T_1}{D_1}. \quad (29)$$

Here $|V_{us}|$ and $|V_{ub}|$ come almost from the mixing in down-type quarks.

It should be noted that the above relations between masses and mixing angles are valid at the unification scale. Of course, we should also keep in mind that these relations are predicted with uncertainty coming from unknown coefficients in the mass matrices.

As shown in Eq. (27), the bottom-tau unification $m_b = m_\tau$ in the simplest SU(5) model holds within a good accuracy even though the mass matrices M_d and M_e^T in Eqs. (20) are different. In addition, the unwanted SU(5) mass relations, $m_\mu = m_s$ and $m_e = m_d$, are avoided in the model, although quarks and leptons are unified into irreducible SU(5) representations. This is because of the gauge charges of matter fields as well as the structure of our vacuum (2). For instance, the Georgi-Jarlskog relation [34], $m_\mu \simeq 3m_s$, can be realized by taking $D_2 \simeq 3T_2$. Moreover, we have a factorization of the mixing angles for quarks,

$$|V_{us}| |V_{cb}| \sim |V_{ub}|, \quad (30)$$

since the quark mixing originates mostly in the structure of the mass matrix for down-type quarks.

All the VEVs of bifundamental fields can be determined only by the mass ratios in Eqs. (28). For example,

without using m_u and m_e , the four mass ratios lead to

$$T_1 \sim \left(\frac{m_c}{m_t}\right)^{-\frac{1}{2}} \left(\frac{m_d}{m_s}\right) \left(\frac{m_\mu}{m_\tau}\right)^{\frac{3}{4}} = 0.12, \quad (31a)$$

$$D_1 \sim \left(\frac{m_c}{m_t}\right)^{\frac{1}{2}} \left(\frac{m_\mu}{m_\tau}\right)^{-\frac{1}{4}} = 0.098, \quad (31b)$$

$$T_2 \sim \left(\frac{m_s}{m_b}\right) \left(\frac{m_\mu}{m_\tau}\right)^{-\frac{1}{2}} = 0.11, \quad (31c)$$

$$D_2 \sim \left(\frac{m_\mu}{m_\tau}\right)^{\frac{1}{2}} = 0.24. \quad (31d)$$

Here we have used fermion masses at the unification scale evaluated in Ref. [35]. From this naive estimation we can say that all these VEVs are typically of order 0.1.

The rest two mass ratios give us the VEV of Φ and a non-trivial mass relation between up-type quarks and charged leptons:

$$\frac{m_c^6}{m_u^4 m_t^2} \sim \frac{m_\mu^5}{m_e^4 m_\tau}. \quad (32)$$

Clearly this relation receives a large correction from unknown coefficients in a very complicated way. Additionally, there are substantial errors in m_u and m_t at the unification scale. However, we may present the charm mass (with the highest power in the relation) avoiding such uncertainties, $m_c \sim 1.1$ GeV, which is consistent with the observation within a factor ~ 3 . The VEV of Φ can be estimated by using m_u

$$\Phi \sim \left(\frac{m_u}{m_t}\right) \left(\frac{m_c}{m_t}\right)^{-\frac{3}{2}} \left(\frac{m_\mu}{m_\tau}\right)^{\frac{3}{4}} = 8.5 \times 10^{-3}, \quad (33)$$

or by using m_e

$$\Phi \sim \left(\frac{m_e}{m_\tau}\right) \left(\frac{m_\mu}{m_\tau}\right)^{-\frac{1}{2}} = 1.1 \times 10^{-3}. \quad (34)$$

We see that these rough estimates give Φ with a large uncertainty.

We have determined all the VEVs from the fermion mass ratios, and hence the CKM matrix elements can be obtained as predictions. Without the uncertainty of Φ , we can evaluate $|V_{cb}|$

$$|V_{cb}| \sim \frac{m_s}{m_b} = 2.7 \times 10^{-2}, \quad (35)$$

which reproduces the experimental data [35] with an error smaller than a factor of two. However, $|V_{us}|$ is obtained

$$|V_{us}| \sim \begin{cases} \frac{m_u m_t^{\frac{3}{2}} m_\mu^{\frac{7}{4}} m_d m_b}{m_c^{\frac{5}{2}} m_\tau^{\frac{7}{4}} m_s^2} = 0.40 \\ \frac{m_t m_e m_\mu^{\frac{1}{2}} m_d m_b}{m_c m_\tau^{\frac{3}{2}} m_s^2} = 0.054 \end{cases}, \quad (36)$$

with a large uncertainty due to the range of Φ in Eqs. (33) and (34). The observed value lies indeed in this range. The factorization relation (30) lead to $|V_{ub}|$ in the range

$$|V_{ub}| \sim (0.0014-0.011). \quad (37)$$

Using the VEV of Φ we can evaluate the effective μ -parameter through Eq. (10)

$$\mu \sim \begin{cases} \left(\frac{m_u}{m_t}\right)^6 \left(\frac{m_c}{m_t}\right)^{-\frac{17}{2}} \left(\frac{m_\mu}{m_\tau}\right)^{\frac{17}{4}} = 89 \text{ TeV} \\ \left(\frac{m_c}{m_t}\right)^{\frac{1}{2}} \left(\frac{m_e}{m_\tau}\right)^6 \left(\frac{m_\mu}{m_\tau}\right)^{-\frac{13}{4}} = 0.54 \text{ GeV} \end{cases}. \quad (38)$$

Although the sixth power of Φ in the expression in Eq. (10) enhances the uncertainty of Φ , the μ -parameter of the weak scale indeed lies in this range. This means that the breaking of the Z_7 symmetry may potentially generate the desired value for μ and the charged fermion mass hierarchies at the same time.

To see this point more clearly, we take the effective μ -term as an input parameter, *e.g.*, $\mu = 100$ GeV. Then, avoiding a large uncertainty Φ becomes from Eq. (31b):

$$\Phi \sim \left(\frac{\mu}{D_1}\right)^{1/6} = 2.7 \times 10^{-3}. \quad (39)$$

In this case the mass ratios for up-quark and electron are estimated as

$$\frac{m_u}{m_t} \sim 2.6 \times 10^{-6}, \quad (40)$$

$$\frac{m_e}{m_\tau} \sim 6.6 \times 10^{-4}, \quad (41)$$

which agree with the observed values

$$\left.\frac{m_u}{m_t}\right|_{\text{obs.}} = 8.1 \times 10^{-6}, \quad (42)$$

$$\left.\frac{m_e}{m_\tau}\right|_{\text{obs.}} = 2.8 \times 10^{-4}, \quad (43)$$

within about a factor of three. Notice that other mass ratios in Eqs. (28) are independent on Φ . We are also able to estimate the CKM elements

$$|V_{us}| \sim 0.13, \quad |V_{cb}| \sim 0.027, \quad |V_{ub}| \sim 0.0034. \quad (44)$$

It is clear that we obtain rather good predictions of the quark mixing angles, which are consistent with the current observation [35] within a factor of two. Therefore, the μ -parameter of the weak scale and the hierarchies of charged fermions both suggest the Z_7 breaking scale given in Eq. (39).

B. Neutrinos

Next, we turn to discuss neutrino masses and mixing. We begin with the Majorana masses for the right-handed

neutrinos. For example, we take here the VEV of Φ to be 2.7×10^{-3} by taking $\mu = 100$ GeV, as shown in Eq. (39). It is then found from Eq. (23) that the Majorana masses are evaluated as

$$M_1 \sim \Phi^3 = 5.0 \times 10^{10} \text{ GeV}, \quad (45)$$

$$M_{2,3} \sim \Phi = 6.7 \times 10^{15} \text{ GeV}. \quad (46)$$

With these superheavy masses the seesaw mechanism works naturally and ensures the smallness of the effective Majorana masses for the light neutrinos which are almost left-handed states [32]. The mass matrix for these left-handed neutrinos is roughly estimated as

$$M_\nu \simeq -M_D^T M_N^{-1} M_D \sim \frac{H_u^2}{\Phi} \begin{pmatrix} \rho^2 & \rho & \rho \\ \rho & 1 & 1 \\ \rho & 1 & 1 \end{pmatrix}, \quad (47)$$

where we defined

$$\rho \equiv \frac{D_1^2 D_2}{\Phi}. \quad (48)$$

Combining Eqs. (31b), (31d) and (39) the parameter ρ turns out to be 0.85, *i.e.*, at most of order one. The obtained result realizes the requirement imposed in Eq. (15).

The particular form of the matrix M_ν given in Eq. (47) leads to interesting consequences in neutrino properties.⁷ First, we introduce the typical scale for neutrino masses m_ν which is given by

$$m_\nu \equiv \frac{H_u^2}{\Phi} \simeq 4.6 \times 10^{-3} \text{ eV}. \quad (49)$$

The numerical estimations, which will be performed below, show that in our model the effective neutrino mass matrix M_ν generates the neutrino masses m_i ($i = 1, 2, 3$) with a small hierarchy $m_3 > m_2 > m_1$. We may then identify the mass squared differences indicated by the atmospheric and solar neutrino oscillations with $\delta m_{\text{atm}}^2 = m_3^2 - m_2^2$ and $\delta m_{\text{sol}}^2 = m_2^2 - m_1^2$. Furthermore, the typical scale m_ν will be found to approximate the second mass m_2 . The recent analysis [37] tells us that $m_2 \simeq \sqrt{\delta m_{\text{sol}}^2} \simeq (7.2\text{--}9.9) \times 10^{-3} \text{ eV}$, which is consistent with the result in Eq. (49) within a factor of two. This result gives us an important bridge between the neutrino masses and the electroweak scale μ -parameter. In fact, we can find from Eq. (10) with $N = 7$ that

$$\mu = \frac{D_1 H_u^{12}}{m_\nu^6}. \quad (50)$$

Thus we can say that if the neutrino masses were smaller or larger by an order of magnitude than the observed values, it would be impossible to get the desired values for

the μ -parameter. Therefore, the neutrino mass scale in the solar neutrino oscillation suggests a strong hint for the μ -parameter of the electroweak scale in our model.

The second consequence from Eq. (47) is that we have a large ν_μ - ν_τ mixing angle solution which is required by the atmospheric neutrino experiments. This is a direct consequence of the fact that 5_2^* and 5_3^* have the same charges under the gauge group G as well as the discrete symmetries in order to cancel the gauge anomalies and also to reproduce the mass hierarchies in down-type quarks and charged leptons. Therefore, the model realizes the lop-sided structure [27, 28, 29, 30].

Finally, we may expect that a large mixing angle for ν_e - ν_μ oscillation as well as a large U_{e3} in the MNS matrix since $\rho = \mathcal{O}(1)$. In fact, one might worry that our model would give the same result as in the so-called ‘‘anarchy’’ model for neutrino masses [38, 39], since the obtained mass matrix in Eq. (47) with $\rho = 1$ is identical to that in the anarchy model. We find, however, the following differences: (i) The hypothesis in the anarchy model, the requirement of basis-independence in neutrino flavor space, is broken in our model, and it holds only between ν_μ and ν_τ , which ensures a large mixing in the atmospheric neutrino oscillations. (ii) Even if $\rho = 1$, there are differences in the Dirac neutrino mass matrix. Namely, as shown in Eq. (25), we have a structure from both right- and left-handed neutrinos. The former one gives a negligible effect, but the latter one is important. Due to the Z_7 charges of 5_i^* the 2-1 and 3-1 elements in M_D are almost zero. Only with these missing elements, there appears a non-trivial structure in M_ν . As we will see below, the numerical estimates show the clear difference from the anarchy model, and in particular there exists the small hierarchy in neutrino mixing angles.

C. Numerical Estimate

As we have shown above, our mass matrices can produce phenomenologically acceptable masses and mixing angles of quarks and leptons including neutrinos with some uncertainty from unknown coefficients of order one. Here we will make a more quantitative estimate by performing the numerical calculations.

First, we include the effects of the RGE (renormalization group equation) evolution. For simplicity, we take the boundary at high energy as Φ where we define all the Yukawa couplings. Then, we solve the one-loop RGEs down to the scale M_Z and compare with the observational data. The threshold corrections from supersymmetric particles are included by the universal scale $M_{\text{SUSY}} = 1 \text{ TeV}$.

We treat unknown coefficients of order one as follows: Our mass matrices contain 20 independent coefficients which are in general complex numbers. Note that some are related of each other due to SU(5) groups. These coefficients are created randomly in three different ways (denoted by the set A, B and C). In all sets the phase

⁷ The neutrino mass matrices in the similar form have been discussed in the literature [36].

TABLE II: VEVs of bifundamental fields and $\tan\beta$ obtained by minimizing χ^2 in Eq. (53). We also show Φ and ρ evaluated from these VEVs.

	T_1	D_1	T_2	D_2	$\tan\beta$	Φ	ρ	χ^2
Set A	0.188	0.103	0.0883	0.284	23.6	0.00271	1.11	0.535
Set B	0.175	0.104	0.0836	0.278	23.8	0.00271	1.10	0.464
Set C	0.159	0.105	0.0768	0.267	24.0	0.00270	1.09	0.404

of each coefficient is created randomly such that it is distributed uniformly in the linear scale within the range $[0, 2\pi]$. The absolute value of each coefficient is created also randomly such that it is distributed uniformly in the logarithmical scale within the range $[1/1.5, 1.5]$ (set A), $[1/2, 2]$ (set B) and $[1/3, 3]$ (set C).

The fermion mass matrices in our model are determined by the six parameters as shown in Eq. (26). In this numerical study we set $\mu = 100$ GeV (at the ultra-violet boundary), for simplicity. There are still five free parameters – T_1 , D_1 , T_2 , D_2 and $\tan\beta$ – which have to be evaluated from the observational data. Here we use the masses for charged fermions m_f and the CKM matrix elements $V_{\alpha\beta}$, and then the neutrino properties are obtained as the outcome.

We create randomly $N = 10^5$ sets of the unknown coefficients, and we calculate m_f^i and $V_{\alpha\beta}^i$ for each set and take the mean values in the logarithmical scale as follows:

$$\overline{m}_f = \exp\left(\frac{1}{N} \sum_{i=1}^N \ln(m_f^i)\right), \quad (51)$$

$$\overline{V}_{\alpha\beta} = \exp\left(\frac{1}{N} \sum_{i=1}^N \ln|V_{\alpha\beta}^i|\right), \quad (52)$$

which should be compared with experimental data. As we will present later, some observables such as the CKM matrix elements are distributed by orders of magnitude, and the mean values in the log-scale are adequate for our analysis.

The five free parameters are fixed by minimizing the function defined by

$$\chi^2 \equiv \sum_{\overline{\mathcal{O}}_I < \mathcal{O}_I^-} \left[\ln\left(\frac{\overline{\mathcal{O}}_I}{\mathcal{O}_I^-}\right) \right]^2 + \sum_{\overline{\mathcal{O}}_I > \mathcal{O}_I^+} \left[\ln\left(\frac{\overline{\mathcal{O}}_I}{\mathcal{O}_I^+}\right) \right]^2, \quad (53)$$

where \mathcal{O}_I denotes m_f or $V_{\alpha\beta}$, and \mathcal{O}_I^+ and \mathcal{O}_I^- the upper and lower limits of the observable with the 3σ error, respectively. In the present analysis we use the masses at the scale M_Z for charged fermions (except for top-quark) estimated in Ref. [35], and the pole mass for top-quark [40]. Further, $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ in Ref. [40] are used for the comparison.

The obtained values for the VEVs and $\tan\beta$ are shown in Table II for the three different sets A, B and C. In Table III we also show the mean values of fermion masses and mixing angles when χ^2 takes its minimal value. We

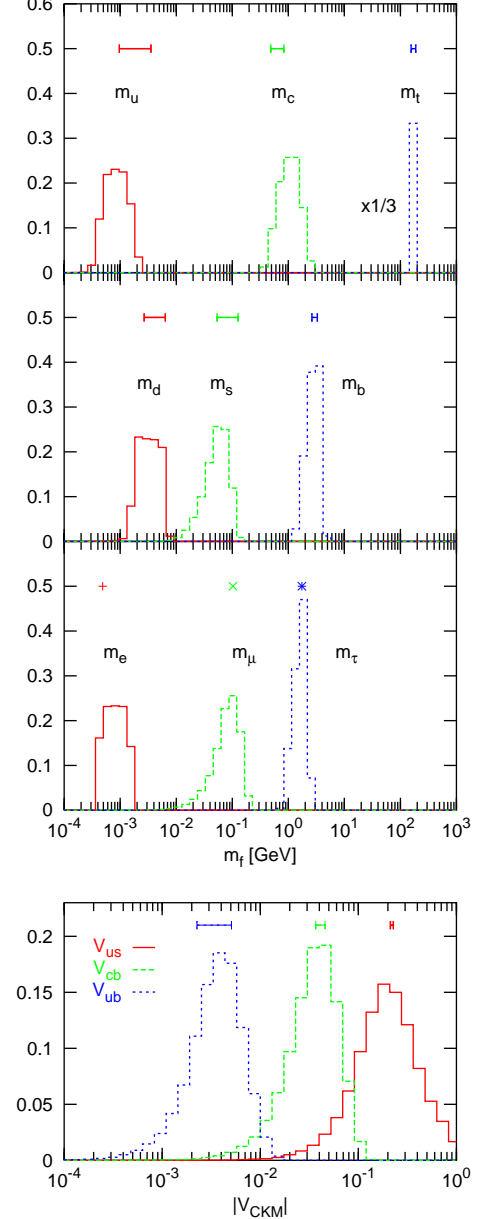


FIG. 1: Distributions of charged fermion masses (top) and the CKM matrix elements (bottom). Distribution is divided by 3 only for top-quark mass. The observational data with the 3σ error are also shown. We use the set B for the unknown coefficients.

see the results do not depend much on the choice of the sets for the unknown coefficients. Henceforth, we will only use the set B for our discussions. Note that the VEVs of bifundamental fields from this numerical estimate are almost consistent with those shown in Eqs. (31). We find $\tan\beta \simeq 24$.

In Figs. 1 we show distributions of the charged fermion masses and the CKM matrix elements by the $N = 10^5$ trials of creating the coefficients. Here only the shape of the distribution should be considered, since the height of

TABLE III: Mean values of fermion masses and mixing angles. We also show the observational data with 1σ error (3σ error) for charged fermion masses and the CKM elements (neutrino mixing angles).

	Observation	Set A	Set B	Set C
m_u [MeV]	$2.33^{+0.42}_{-0.45}$	0.859	0.857	0.858
m_c [MeV]	677^{+56}_{-61}	1.06	1.05	1.07
m_t [GeV]	174 ± 5.5	181	179	174
m_d [MeV]	$4.69^{+0.60}_{-0.66}$	3.71	3.25	2.69
m_s [MeV]	$93.4^{+11.8}_{-13.0}$	51.3	51.2	50.2
m_b [GeV]	3.00 ± 0.11	2.68	2.75	2.91
m_e [MeV]	0.487	0.832	0.802	0.760
m_μ [MeV]	103	72.2	74.4	74.9
m_τ [GeV]	1.75	1.53	1.57	1.68
$ V_{us} $	0.220 ± 0.0026	0.187	0.187	0.187
$ V_{cb} $	$(4.13 \pm 0.15) \times 10^{-2}$	3.24×10^{-2}	3.26×10^{-2}	3.33×10^{-2}
$ V_{ub} $	$(3.67 \pm 0.47) \times 10^{-3}$	3.47×10^{-3}	3.37×10^{-3}	3.07×10^{-3}
$\tan^2 \theta_{23}$	0.49–2.2	1.04	1.04	1.05
$\tan^2 \theta_{12}$	0.29–0.64	0.408	0.350	0.280
$\tan^2 \theta_{13}$	< 0.0571	0.118	0.0954	0.0608
m_1 [eV]	—	4.02×10^{-4}	3.63×10^{-4}	2.61×10^{-4}
m_2 [eV]	—	3.11×10^{-3}	3.09×10^{-3}	3.08×10^{-3}
m_3 [eV]	—	1.28×10^{-2}	1.39×10^{-2}	1.69×10^{-2}
M_1 [GeV]	—	0.911×10^{11}	0.965×10^{11}	1.14×10^{11}
M_2 [GeV]	—	4.36×10^{15}	4.84×10^{15}	5.34×10^{15}
M_3 [GeV]	—	1.23×10^{16}	1.30×10^{16}	1.47×10^{16}

the histogram reflects just the bin size of the horizontal axis. The spread of these distributions of masses and mixing angles originates in our ignorance of the hidden physics to determine the unknown coefficients.

We can see that our model predicts – of order of magnitude wise – all masses of charged fermions as well as all quark mixing angles by using only five parameters (apart from $\mu = 100$ GeV). One may worry that the model induces the wrong values for masses and mixing angles with some possibility, since they are obtained with the distributions, *e.g.*, $|V_{us}|$ distributed by two orders of magnitude. However, the peaks of the distributions tell that the observed values are preferred in the model. In this sense, the model can reproduce the charged fermion mass spectra and flavor mixing rather well.

Now let us discuss the neutrino masses and mixing. These quantities are obtained as the prediction, since we have determined all free parameters from charged fermion properties. The distributions of left-handed neutrino masses m_i are shown in Fig. 2. It is found that there exists the small hierarchy in neutrino masses (compared with charged fermions). This feature comes from the same reason as in the anarchy model, *i.e.*, the multiplication of three mass matrices in the seesaw formula (24) makes eigenvalues scattered and it is possible to have $(m_3/m_2) \sim 10$ [38, 39]. The peaks of the distributions are located at $m_1 \simeq 5 \times 10^{-4}$ eV, $m_2 \simeq 3 \times 10^{-3}$ eV

and $m_3 \simeq 1 \times 10^{-2}$ eV.⁸ This result shows that the most preferred value of m_2 corresponds to the typical neutrino mass scale m_ν (49):

$$m_\nu = \frac{H_u^2}{\Phi} = 4.6 \times 10^{-3} \text{ eV} \sim m_2, \quad (54)$$

where we have used the fitted values for Φ and $\tan \beta$ in Table II.

From the small hierarchy in the neutrino masses m_i the mass squared differences in the atmospheric and solar neutrino oscillations are given by [37]

$$\sqrt{\delta m_{\text{atm}}^2} = (3.7\text{--}5.8) \times 10^{-2} \text{ eV} \simeq m_3, \quad (55)$$

$$\sqrt{\delta m_{\text{sol}}^2} = (7.2\text{--}9.9) \times 10^{-3} \text{ eV} \simeq m_2. \quad (56)$$

We see that the most preferred values for m_3 and m_2 are consistent with the observation within a factor of three or so. With this respect, our model is successful to explain the observed mass spectra of neutrinos.⁹

⁸ We observe numerically that the peak locations of m_1 and m_3 (but not m_2) depend slightly on the choice of the unknown coefficients. When we take the wider range of their norm, the peaks of m_1 and m_3 are shifted to the smaller and larger values, respectively. The differences, however, are within a factor of two.

⁹ Although the factors of three or so are beyond our approximation

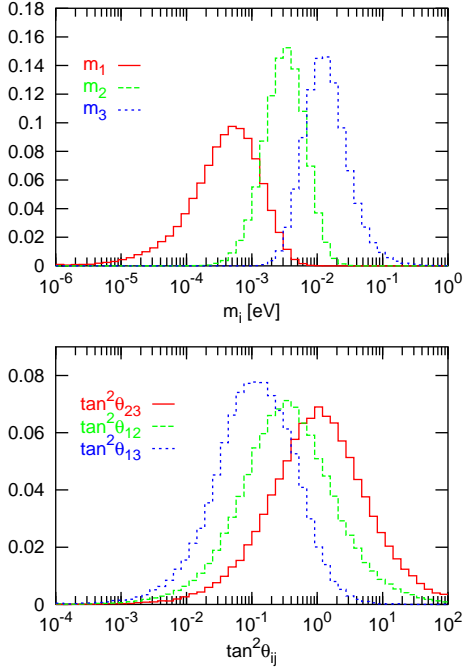


FIG. 2: Distributions of left-handed neutrino masses m_i (top) and neutrino mixing angles $\tan^2 \theta_{ij}$ (bottom). We use the set B for the unknown coefficients.

In Fig. 2 we show also the distributions of neutrino mixing angles, $\tan^2 \theta_{ij}$. Although the distributions are spread by a few orders of magnitude, we may find characteristic features in our model. First, similar to the neutrino masses, we find a small hierarchy in the neutrino mixing angles, $\tan^2 \theta_{23} \gtrsim \tan^2 \theta_{12} \gtrsim \tan^2 \theta_{13}$, compared with the CKM angles. We should stress that this hierarchy is obtained even when $\rho = 1.1$ [see Eq. (47)]. This is because of the missing elements in the Dirac neutrino mass matrix in Eq. (25).

The peak of the $\tan^2 \theta_{23}$ distribution suggests that it is the maximal mixing which is confirmed in the atmospheric neutrino oscillations.¹⁰ This is the direct consequence of the fact that the matter fields, 5_2^* and 5_3^* , carry the same charges for the product group G as well as the discrete symmetry Z_7 , as mentioned before. The peak location of the $\tan^2 \theta_{12}$ distribution is found to be $\tan^2 \theta_{12} = 0.35$, and hence it prefers large values, rather than the maximal angle. This is completely different from the anarchy model where the distributions of $\tan^2 \theta_{23}$ and $\tan^2 \theta_{12}$ are the same.

in this analysis, we may correct them just by changing the overall normalization of M_D or M_N with a factor of $\sqrt{3}$ or $1/3$, which leaves other results unchanged.

¹⁰ It is found numerically that the peak locations of $\tan^2 \theta_{12}$ and $\tan^2 \theta_{13}$ (but not $\tan^2 \theta_{23}$) depend slightly on the choice of the unknown coefficients. The wider range for the norm of the coefficients makes the peaks of $\tan^2 \theta_{12}$ and $\tan^2 \theta_{13}$ smaller. The differences, however, are within a factor of two.

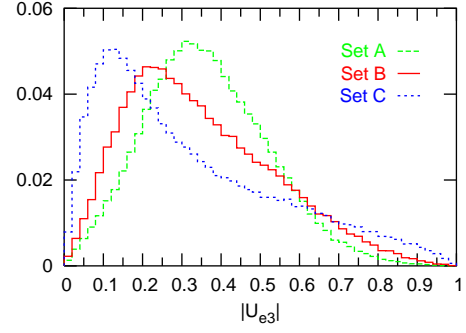


FIG. 3: Distributions of $|U_{e3}|$ in the neutrino mixing matrix. We use the set A, B and C for the unknown coefficients.

Moreover, it is interesting to mention that in our model $\tan^2 \theta_{13} < \tan^2 \theta_{12}$ is preferred, and the peak location of the $\tan^2 \theta_{13}$ distribution is found to be $\tan^2 \theta_{13} = 0.1$, which is consistent with the current limit $\tan^2 \theta_{13} < 5.7 \times 10^{-2}$ [37] within a factor of two. To be specific, we show in Fig. 3 the distributions of the $|U_{e3}|$ element in the MNS matrix for the three different sets, A, B and C. The most preferred value for $|U_{e3}|$ is found to be 0.1–0.3 depending on the set. Note that these distributions are plotted in the linear horizontal axis, and then we may easily see the difference between the three sets. These results show that the forthcoming experiments on $|U_{e3}|$ are crucial for our model.

To summarize, our model describes well the masses of quarks and charged leptons and also the CKM mixing angles when $\mu = 100$ GeV. Inversely, the observed mass spectra require the μ -parameter around the weak scale. In addition, the model predicts the desired features of neutrino properties, the correct mass scale and the small hierarchies in both masses and mixing angles. We should stress again that the typical neutrino mass scale (the solar neutrino mass scale) also indicates the weak scale μ independently on the charged fermion sector.

V. LEPTOGENESIS

As studied above, we have determined all the Yukawa couplings (or mass matrices) of quarks and leptons including neutrinos, and hence we may calculate various quantities in the flavor physics. As an example, we discuss here the implication of the model to the leptogenesis mechanism [41] for the cosmic baryon asymmetry.

The non-equilibrium decays of right-handed Majorana neutrinos can generate the lepton asymmetry, if the CP symmetry is broken in neutrino sector, because their mass terms break the lepton number. The CP asymmetry in the decay of right-handed neutrino (here we consider only the lightest one) is parameterized by ϵ_1 , which is calculated from the interference between the tree and

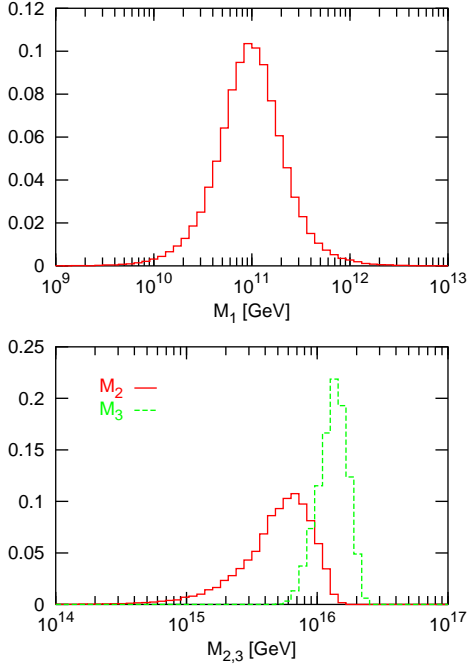


FIG. 4: Distributions of Majorana masses for right-handed neutrinos, M_1 (top), M_2 and M_3 (bottom). We use the set B for the unknown coefficients.

one-loop amplitudes for the decay processes [42, 43, 44]:

$$\epsilon_1 \simeq \frac{1}{8\pi(Y_\nu Y_\nu^\dagger)_{11}} \sum_{i=2,3} \text{Im} \left[\{(Y_\nu Y_\nu^\dagger)_{1i}\}^2 \right] f(x_i), \quad (57)$$

where $x_i = M_i^2/M_1^2$ and $f(x_i)$ is defined by

$$f(x_i) \equiv -\sqrt{x_i} \ln \left(1 + \frac{1}{x_i} \right) - \frac{2\sqrt{x_i}}{x_i - 1}. \quad (58)$$

Here and hereafter we work in the base of the Majorana mass matrix, Eq. (23), for right-handed neutrinos being diagonal. The distributions of the Majorana masses are displayed in Fig. 4. We find that typical masses are $M_1 \simeq 10^{11}$ GeV, $M_2 \simeq 7 \times 10^{15}$ GeV and $M_3 \simeq 1.3 \times 10^{16}$ GeV, *i.e.*, $M_1 \ll M_2 \lesssim M_3$.

Since we have already obtained the Dirac Yukawa couplings for neutrinos, we are able to calculate the CP asymmetry parameter ϵ_1 . We should note that since all the phases of the unknown coefficients are distributed randomly in the range of $[0, 2\pi]$, the sign of ϵ_1 parameter can be positive or negative. In this analysis we neglect the sign of ϵ_1 and discuss only its absolute value. The distribution of $|\epsilon_1|$ at the scale of M_1 can be found in Fig. 5: $|\epsilon_1|$ is distributed by many orders of magnitude, however, it is seen that the most preferred value is $|\epsilon_1| \simeq 2 \times 10^{-6}$.

This lepton asymmetry is produced when the cosmic temperature is $T \sim M_1$ and it is partially converted into the baryon asymmetry through the electroweak sphaleron [45]. The baryon-to-entropy ratio of

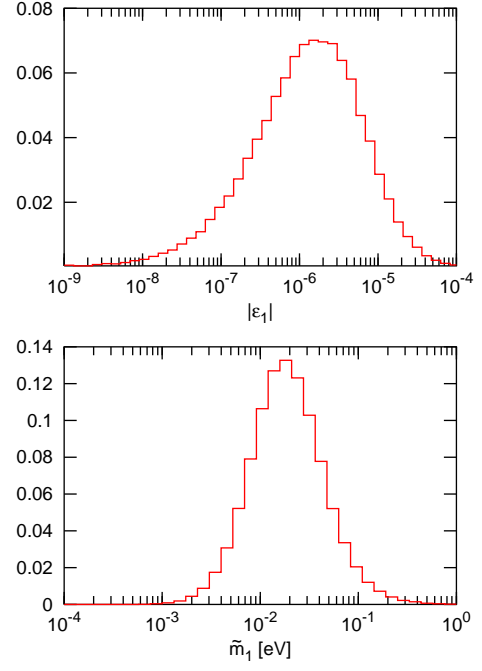


FIG. 5: Distributions of the CP asymmetry parameter $|\epsilon_1|$ for the lightest right-handed neutrino (top) and the effective neutrino mass \tilde{m}_1 (bottom). We use the set B for the unknown coefficients.

the present universe is given by [41, 42, 46, 47]

$$\frac{n_B}{s} = 1.5 \times 10^{-3} \kappa |\epsilon_1|, \quad (59)$$

where κ is the efficiency factor, which should be estimated by solving the Boltzmann equations. This factor depends on the lightest Majorana mass M_1 and the effective neutrino mass \tilde{m}_1 which is defined by [46]

$$\tilde{m}_1 = \frac{(M_D M_D^\dagger)_{11}}{M_1}. \quad (60)$$

Note that M_D is the Dirac neutrino mass matrix in the basis where right-handed neutrino mass matrix is diagonal.

Fig. 5 shows also the distribution of \tilde{m}_1 in our model, and we find $\tilde{m}_1 = \mathcal{O}(10^{-2})$ eV. According to Ref. [47] when $M_1 = 10^{11}$ GeV and $\tilde{m}_1 = 2 \times 10^{-2}$ eV, κ becomes about 10^{-2} . Therefore, the typical values $|\epsilon_1| = 2 \times 10^{-6}$ and $\kappa = 10^{-2}$ induce the baryon asymmetry

$$\frac{n_B}{s} \sim 3 \times 10^{-11}, \quad (61)$$

which is consistent with the recent observation $(n_B/s)_{\text{obs.}} = (9.0 \pm 0.4) \times 10^{-11}$ [40] within a factor three or so. However, the discrepancy of a factor three is of course beyond our estimation of order of magnitude wise, and also there is a sizable theoretical uncertainty

in the efficiency factor [46, 47].¹¹

This is a very encouraging result of the model, *i.e.*, the observed cosmic baryon asymmetry is naturally explained (with some uncertainty from the unknown coefficients) by invoking the leptogenesis mechanism. Unfortunately, the sign of the asymmetry is not determined in our approach. To realize this successful scenario, the maximal temperature of the universe should be higher than $M_1 \simeq 10^{11}$ GeV so that the lightest right-handed neutrinos were thermalized. Such high temperatures may lead to the cosmological gravitino problem in the interesting region of the gravitino mass (of the weak scale) [48, 49]. To avoid this difficulty we may go into the heavy gravitino masses of $\mathcal{O}(100)$ TeV suggested from the anomaly mediated supersymmetry breaking mechanism [50].¹² (See the discussion in Ref. [52].)

VI. CONCLUSIONS

We have investigated in this article a supersymmetric model based on the gauge group $SU(5) \times SU(5) \times SU(5)$ with the discrete symmetries $Z_7 \times Z_2$. The gauge symmetry is broken into the standard model group only by using the bifundamental Higgs fields without introducing the adjoint or higher-dimensional Higgs fields. This point gives us a promising connection to the string theories. In fact, all the fields in the model are available in the string spectrum with the affine level one. The mass splitting between the weak-doublets and the color-triplets of Higgs fields is ensured by the Z_7 symmetry. The effective μ -parameter is then generated by its breaking together with the gauge symmetry.

We have constructed the model for realistic masses and mixing of quarks and leptons including neutrinos. The hierarchical structures in the Yukawa couplings originate in the breaking pattern of the gauge and discrete symmetries. The fermion mass matrices are determined by only the six parameters – T_1 , D_1 , T_2 , D_2 , Φ and $\tan \beta = H_u/H_d$ – which are all the VEVs of the Higgs fields introduced in the model. All the masses and mixing angles are predicted being consistent with the observation including the uncertainty from the unknown coefficients of order unity, say, within a factor of two or three.

There have been obtained several interesting features: First, the bottom-tau unification, $m_b \simeq m_\tau$, is realized within a good accuracy while avoiding the unwanted $SU(5)$ mass relations, $m_s = m_\mu$ and $m_d = m_e$. Second, right-handed neutrinos carry non-trivial charges of

the discrete Z_7 symmetry for the doublet-triplet splitting, and its breaking generates the superheavy Majorana masses. Thus, the model offers the natural framework for the seesaw mechanism. Third, the Z_7 breaking gives us a non-trivial bridge between three independent facts, (i) the μ -parameter, (ii) the hierarchies in the charged fermion masses and the quark-mixing angles, and (iii) the typical neutrino mass m_ν (which corresponds to the solar neutrino mass scale). It has been shown that both of the observational data on (ii) and (iii) point toward the Z_7 breaking scale $\Phi \simeq 2.7 \times 10^{-3} = 6.6 \times 10^{15}$ GeV, which suggests interestingly the μ -parameter of order of the weak scale. Finally, the particular structure in the Dirac neutrino mass matrix (25) generates the small hierarchy in the neutrino mixing angles $\tan^2 \theta_{23} \gtrsim \tan^2 \theta_{12} \gtrsim \tan^2 \theta_{13}$. The most preferred values of the mixing angles indicate that the atmospheric and solar neutrino-mixing are maximal and large, respectively. The element $|U_{e3}|$ in the MNS matrix is typically $|U_{e3}| \sim 0.1\text{--}0.3$, which means that the present model will be tested by the future experiments on U_{e3} .

We have also discussed the implication of the model to the baryon asymmetry of the universe, *i.e.*, the leptogenesis mechanism by the decay of the lightest right-handed neutrino which is produced thermally in the early universe. Since the model has fixed all the couplings and masses of neutrino (including the Majorana masses for right-handed neutrinos), we may calculate of order of magnitude the baryon asymmetry without any further assumptions. Due to the random phases of the unknown coefficients the sign of the asymmetry cannot be determined. It has been shown that the prediction of the model (obtained as the most preferred value) agrees with the observation within a factor three or so, and hence the model naturally accounts for the baryon asymmetry of the present universe. For this successful scenario the highest temperature of the universe $T \gtrsim M_1 \simeq 10^{11}$ GeV is required, and the cosmological gravitino problem should be avoided somehow.

Before closing this article, we would like to give some comments: It has been found from the fermion masses that the gauge symmetry breaking scales, namely the VEVs of the bifundamental fields, are $0.08\text{--}0.3 \simeq (2\text{--}5) \times 10^{17}$ GeV, which is about one order higher than the unification scale $\simeq 2 \times 10^{16}$ GeV. This might arise a trouble in the gauge coupling unification, although it holds at the tree-level. However, we have to take into account the threshold corrections from superheavy particles beyond the minimal supersymmetric standard model. In fact, such particles are indeed present in our model and may potentially give rise to the coupling unification. Moreover, we keep in mind that there might exist non-negligible corrections from the physics beyond our model, *e.g.*, the string theories. We should also mention that the breaking of our gauge and discrete symmetries generates the suppression for the Yukawa couplings for the color-triplet Higgs fields and also for the supersymmetry-breaking masses for the scalar particles. Therefore, our

¹¹ The correct value for the baryon asymmetry may be obtained by changing the overall normalization of M_D (or Y_D) with a factor of $\sqrt{3}$, which also corrects the discrepancy in the light neutrino masses as mentioned in the footnote 9.

¹² In Ref. [51] a possibility of having high temperatures has been discussed.

model would avoid the rapid proton decay and also the supersymmetric flavor problems. Details of these issues will be discussed elsewhere [53].

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